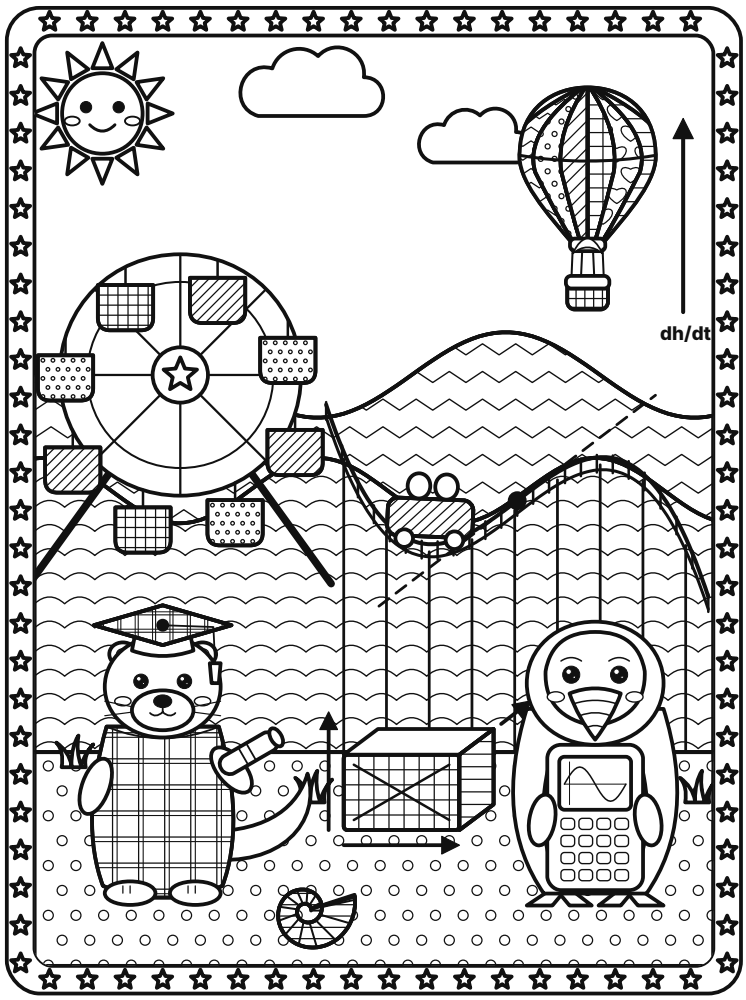
REAL-LIFE MATHS

Higher Mathematics

Revision Guide

Notes, worked examples, exam-style questions and two practice papers, with full answers



C. Blewis

Blewis LEARNING

Contents

GETTING STARTED		
	How to use this guide	4
	The exam	5
	Exam skills	6
	Formulae	8
ALGEBRA AND FUNCTIONS		
1.1	Polynomials	11
1.2	Quadratics	14
1.3	Functions	18
1.4	Graphs and transformations	22
1.5	Logarithms and exponentials	26
1.6	Recurrence relations	30
TRIGONOMETRY		
2.1	Trigonometric formulae and identities	34
2.2	The wave function	39
2.3	Solving trigonometric equations	43
GEOMETRY AND VECTORS		
3.1	The straight line	48
3.2	Circles	51
3.3	Vectors in three dimensions	55
3.4	The scalar product	59
CALCULUS		
4.1	Differentiation	63
4.2	Using differentiation with curves	66
4.3	Applications of differentiation	71
4.4	Integration	76
4.5	Definite integrals and area	80
PRACTICE AND ANSWERS		
	Practice exam 1 (Papers 1 and 2)	84
	Practice exam 2 (Papers 1 and 2)	90
	Answers	96
	Practice exam marking	106

How to use this guide

This guide covers the whole Higher Mathematics course in four areas, with reasoning built in throughout. Each key area follows the same pattern, so you always know what comes next.

EACH KEY AREA HAS

Notes	the ideas you need, in short sections with diagrams
Methods	each skill with a clear method and a worked example set out step by step
Exam tips	common mistakes and how to pick up every mark
Quick check	short questions to test yourself straight away
Exam-style questions	longer questions with marks, like the real exam

A GOOD WAY TO REVISE

1. Read the notes and cover the worked example. Can you do it yourself?
2. Try the quick check without looking back. Mark it using the answers at the back.
3. Do the exam-style questions in your jotter, with full working. Then mark them carefully against the marking guidance.
4. Tick the key area off on the tracker at the start of each area: **1** need more work, **2** getting there, **3** confident.
5. When you have revised everything, sit the practice papers under exam conditions.

ANSWERS

All answers are at the back. For exam-style questions the marking guidance shows where each mark comes from, so you can see exactly what examiners look for.

CHECK THE COURSE

Exam arrangements and course content can change. Your teacher and the latest course documents always come first.

The exam

Higher Mathematics is assessed by two question papers, usually sat on the same day. There is no coursework. **Arrangements can change, so check the latest arrangements with your teacher.**

Paper	What it is	Time	Marks
Paper 1	**Non-calculator.** Short and extended questions across the course.	1 h 15 min	55
Paper 2	**Calculator allowed.** Short and extended questions, more problem solving.	1 h 30 min	65
Total	A formulae list is provided in both papers (see the Formulae pages).		120

Command words

Command word	What you need to do
Find, calculate, evaluate, determine	work out the answer and show the working that leads to it
Show that	the answer is given, so every step must be shown and the last line must match it exactly
Hence	use the result you have just found. Another method usually gets no marks
Hence, or otherwise	using the last result is quickest, but any valid method is accepted
Sketch	a neat graph, not to scale, with the key features labelled: roots, intercepts, turning points, asymptotes
Justify, explain	give a reason in words that refers to your working, e.g. “the discriminant is negative, so…”
Prove	a complete logical argument from known facts to the result, with every step stated
Express … in the form	rewrite in exactly the form asked, e.g. $k\cos(x - a)$ or $a(x + p)^2 + q$

ON THE DAY

- Both papers allow about **1.4 minutes per mark**. Leave time to check.
- Show all working. Most marks are for method, and a slip early on does not stop you earning later marks.
- In Paper 2, check your calculator is in **radians** or **degrees** to match the question before any trigonometry.

Exam skills

How marks are given

Each mark is for one clear step. Markers look for evidence of that step in your working, so a correct answer with no working can score very little. Marks fall into three kinds:

Strategy	choosing a method that works, e.g. “set the derivative to zero” or “use the discriminant”
Process	carrying out each step correctly: differentiating, substituting, solving, simplifying
Communication	stating the result clearly: the nature of a turning point, the final equation, a reason in words

HOW A 6-MARK ANSWER IS MARKED

Find the coordinates of the stationary points on $y = x^3 - 3x^2 + 4$ and determine their nature.

Process	$\frac{dy}{dx} = 3x^2 - 6x$ (1)
Strategy	$3x^2 - 6x = 0$ (1)
Process	$3x(x - 2) = 0$, so $x = 0$ or $x = 2$ (1)
Process	$y = 4$ and $y = 0$, so (0, 4) and (2, 0) (1)
Process	nature table or second derivative: $6x - 6$ is -6 at $x = 0$ and 6 at $x = 2$ (1)
Communication	**(0, 4) is a maximum turning point; (2, 0) is a minimum turning point** (1)

> **EXAM TIP**
> If you make a slip, keep going. Markers follow your working through, so later marks can still be awarded if the method is right. Do not score out working unless you have replaced it.

Working to show

- Write the line where you set something equal to zero, e.g. “$\frac{dy}{dx} = 0$”, or “at intersection, $x^2 + 1 = 3x - 1$”.
- Show a substitution before the answer: $f(2) = 2^3 - 3(2) + 1 = 3$.
- For a definite integral, write the square bracket line before you substitute the limits.
- In “show that” questions, the final line must be the given result, word for word.

1.1 Polynomials

Solving cubic and quartic equations

Factorise fully, then set each factor equal to zero. A quartic usually needs two synthetic divisions.

−1	**1**	**−2**	**−7**	**8**	**12**
		−1	3	4	−12
	1	**−3**	**−4**	**12**	**0**

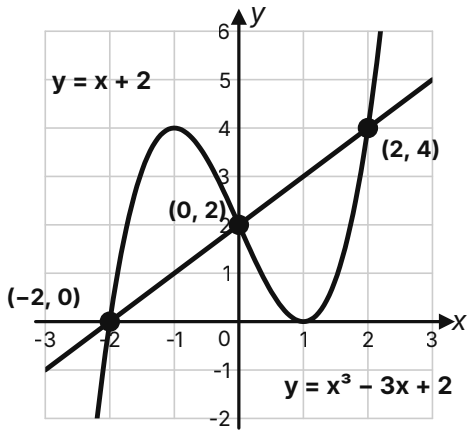
2	**1**	**−3**	**−4**	**12**
		2	−2	−12
	1	**−1**	**−6**	**0**

WORKED EXAMPLE

Solve $x^4 - 2x^3 - 7x^2 + 8x + 12 = 0$.

Try x = −1	remainder 0, so (x + 1) is a factor; quotient $x^3 - 3x^2 - 4x + 12$
Try x = 2	remainder 0, so (x − 2) is a factor; quotient $x^2 - x - 6$
Factorise	$(x + 1)(x - 2)(x - 3)(x + 2) = 0$
Answer	**x = −2, −1, 2, 3**

Where curves meet



WORKED EXAMPLE

Find where $y = x^3 - 3x + 2$ meets $y = x + 2$.

Equate	$x^3 - 3x + 2 = x + 2$
Rearrange	$x^3 - 4x = 0$, so $x(x - 2)(x + 2) = 0$
x values	x = −2, 0, 2
Points	**(−2, 0), (0, 2), (2, 4)**

EXAM TIP

A repeated root when you equate a line and a curve means the line is a **tangent** to the curve at that point.

1.1 Polynomials

Quick check

1 Show that $(x - 1)$ is a factor of $x^3 - 6x^2 + 11x - 6$ and factorise fully.

2 Find the remainder when $2x^3 - x^2 + 4$ is divided by $(x - 2)$.

3 Factorise fully $x^3 + 2x^2 - 5x - 6$.

4 Solve $2x^3 - 3x^2 - 3x + 2 = 0$.

5 $(x + 3)$ is a factor of $x^3 + 2x^2 + px + 6$. Find p.

6 Find the points where $y = x^3$ and $y = 4x$ meet.

Exam-style questions (17 marks)

1 $f(x) = x^3 - 4x^2 + x + 6$. **(non-calculator)**

(a) Show that $(x - 2)$ is a factor of $f(x)$. **(2)**

(b) Hence factorise $f(x)$ fully. **(2)**

(c) Solve $f(x) = 0$. **(1)**

2 $(x + 1)$ and $(x - 2)$ are both factors of $2x^3 + ax^2 + bx - 6$. Find the values of a and b. **(non-calculator)**

(4)

3 A curve has equation $y = x^3 - 2x^2 - 5x + 6$ and a line has equation $y = 6 - 5x$.

(a) Show that the x-coordinates of the points where the line and curve meet satisfy $x^3 - 2x^2 = 0$. **(2)**

(b) Hence find the coordinates of the points of intersection. **(2)**

(c) Explain what the repeated root tells you about the line at one of these points. **(1)**

4 Leo says that the equation $x^3 + x^2 + 2x + 2 = 0$ has three real roots. Show that $(x + 1)$ is a factor of $x^3 + x^2 + 2x + 2$ and hence show that Leo is wrong.

(3)

1.2 Quadratics

A quadratic $f(x) = ax^2 + bx + c$ has a parabola as its graph. In **completed square form** $f(x) = a(x + p)^2 + q$ the turning point is $(-p, q)$. If $a > 0$ it is a minimum; if $a < 0$ it is a maximum.

KEY POINTS

- Take out a as a factor from the x^2 and x terms **before** completing the square.
- $x^2 + bx = (x + \frac{b}{2})^2 - (\frac{b}{2})^2$
- The axis of symmetry is $x = -p$.
- Since $(x + p)^2 \geq 0$, the least (or greatest) value of $f(x)$ is q.

Completing the square with a non-unit x^2 coefficient

WORKED EXAMPLE

Write $2x^2 - 12x + 7$ in the form $a(x + p)^2 + q$ and state the turning point.

Factor out 2	$2(x^2 - 6x) + 7$
Complete square	$2[(x - 3)^2 - 9] + 7$
Simplify	$2(x - 3)^2 - 18 + 7 =$ **$2(x - 3)^2 - 11$**
Turning point	**$(3, -11)$**, a minimum since $2 > 0$

WORKED EXAMPLE

Write $-x^2 + 4x + 1$ in the form $a(x + p)^2 + q$.

Factor out −1	$-(x^2 - 4x) + 1$
Complete	$-[(x - 2)^2 - 4] + 1$
Answer	**$-(x - 2)^2 + 5$**, maximum $(2, 5)$

WORKED EXAMPLE

Write $3x^2 + 12x + 5$ in the form $a(x + p)^2 + q$.

Factor out 3	$3(x^2 + 4x) + 5$
Complete	$3[(x + 2)^2 - 4] + 5$
Answer	**$3(x + 2)^2 - 7$**, minimum $(-2, -7)$

EXAM TIP

Check by expanding: $2(x - 3)^2 - 11 = 2x^2 - 12x + 18 - 11 = 2x^2 - 12x + 7$. It takes ten seconds and catches sign slips.

1.2 Quadratics

The discriminant

For $ax^2 + bx + c = 0$ the **discriminant** is $b^2 - 4ac$. It tells you the nature of the roots without solving.

$b^2 - 4ac$	Roots	Graph
> 0	two real and distinct	crosses the x-axis twice
$= 0$	real and equal (one repeated root)	touches the x-axis
< 0	no real roots	does not meet the x-axis
≥ 0	real roots	meets the x-axis

WORKED EXAMPLE

$kx^2 + 4x + 1 = 0$ has equal roots. Find k.

Equal roots $b^2 - 4ac = 0$

Substitute $16 - 4k = 0$

Answer **k = 4**

WORKED EXAMPLE

$2x^2 + 4x + p = 0$ has no real roots. Find the range of p.

No real roots $b^2 - 4ac < 0$

Substitute $16 - 8p < 0$

Answer **p > 2**

Tangency

To test whether a line meets a curve, substitute the line into the curve to get a quadratic. Then use its discriminant: equal roots means the line is a **tangent**; no real roots means the line misses the curve.

WORKED EXAMPLE

Find c so that $y = 2x + c$ is a tangent to $y = x^2 + 4x + 3$, and find the point of contact.

Equate $x^2 + 4x + 3 = 2x + c$, so $x^2 + 2x + (3 - c) = 0$

Tangent $b^2 - 4ac = 0$: $4 - 4(3 - c) = 0$, so $4c = 8$ and **c = 2**

Solve $x^2 + 2x + 1 = 0$, so $(x + 1)^2 = 0$ and $x = -1$

Point $y = 2(-1) + 2 = 0$, so the point of contact is **(−1, 0)**

EXAM TIP

State the condition in words before you substitute: "for tangency, $b^2 - 4ac = 0$". This is often a mark on its own.

FREE SAMPLE

Higher Mathematics Revision Guide

This sample shows a few pages. The full printed workbook is available from Amazon.

blewislearning.co.uk

Blewis
LEARNING