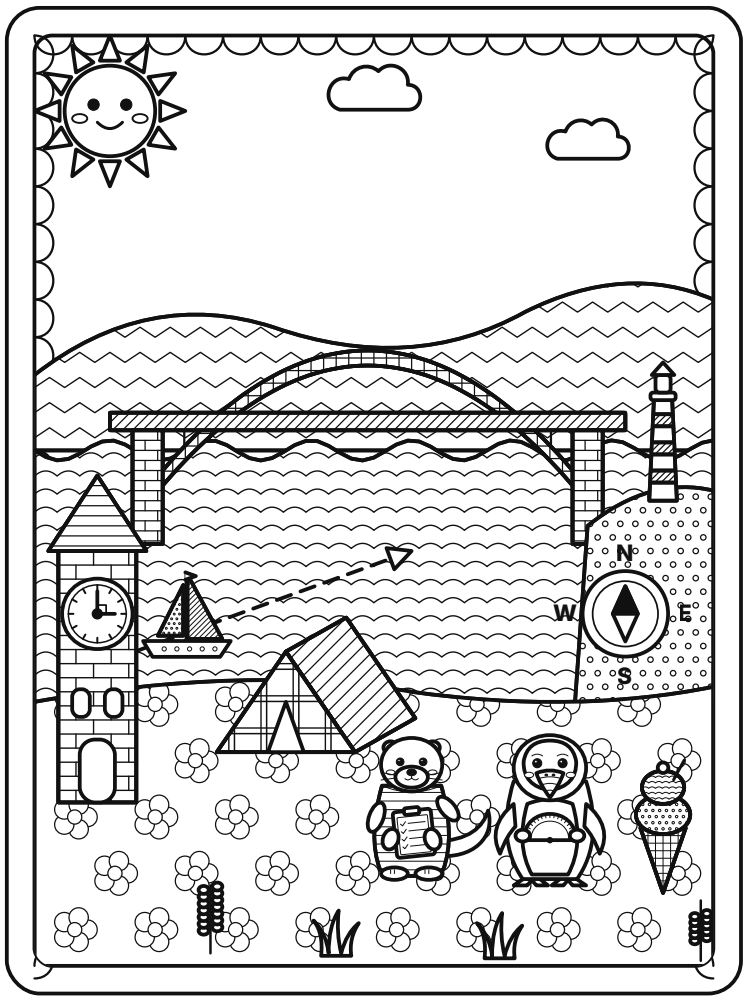
REAL-LIFE MATHS

National 5 Mathematics

Revision Guide

Notes, worked examples, exam-style questions and two practice papers, with full answers



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Blewis LEARNING

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How to use this guide

This guide covers the whole National 5 Mathematics course in five skill areas, with reasoning built in throughout. Each key area follows the same pattern, so you always know what comes next.

EACH KEY AREA HAS

Notes	the ideas you need, in short sections with diagrams
Methods	each skill with a clear method and a worked example set out step by step
Exam tips	common mistakes and how to pick up every mark
Quick check	short questions to test yourself straight away
Exam-style questions	longer questions with marks, like the real exam

A GOOD WAY TO REVISE

1. Read the notes and cover the worked example. Can you do it yourself?
2. Try the quick check without looking back. Mark it using the answers at the back.
3. Do the exam-style questions in your jotter, with full working. Then mark them carefully against the marking guidance.
4. Tick the key area off on the tracker at the start of each area: **1** need more work, **2** getting there, **3** confident.
5. When you have revised everything, sit the practice papers under exam conditions.

ANSWERS

All answers are at the back. For exam-style questions the marking guidance shows where each mark comes from, so you can see exactly what examiners look for.

CHECK THE COURSE

Exam arrangements and course content can change. Your teacher and the latest course documents always come first.

The exam

National 5 Mathematics is assessed by two question papers sat on the same day. There is no coursework. **Arrangements can change, so check the latest arrangements with your teacher.**

Paper	What it is	Time	Marks
Paper 1	**Non-calculator.** Short and longer questions across the whole course.	1 hour	40
Paper 2	**Calculator allowed.** Short and longer questions, more problem solving in context.	1 hour 30 minutes	50
Total	Both papers include a formulae list (see the Formulae page).		90

Command words

Command word	What you need to do
Calculate, find, evaluate	work out the answer; show your working
Simplify	write in the simplest form, e.g. collect like terms, cancel, simplify surds
Factorise	write as a product of factors (brackets)
Solve	find the value(s) of the unknown that make the equation or inequation true
Show that	the answer is given: every step must be shown, and the final line must match
Hence	use the answer you have just found to do the next part
Determine	find, often by more than one step or from a graph or diagram
Sketch	a neat diagram, not to scale, showing the key features (roots, turning point, intercepts)
Explain, justify, give a reason	a reason in words using your working or the context: “yes, because…”
Comment on	compare and describe what the numbers mean in the context

ON THE DAY

- Paper 1: about **1.5 minutes per mark**; Paper 2: about **1.8 minutes per mark**. Leave time to check.
- Read the whole question. Underline what you are asked to find and any rounding instruction.
- Show all working. A wrong final answer with clear working can still get most of the marks.
- In Paper 2, check your calculator is in **degrees** before any trigonometry.

Exam skills

How marks are given

Markers give marks for each stage of your answer, not just the final answer. In a longer question there are usually marks for:

Strategy	choosing a method that will work, e.g. using Pythagoras then trigonometry, or setting up an equation
Process	carrying out each step correctly: substituting, expanding, solving, calculating
Communication	stating the answer clearly with units, rounding as asked, and answering the question in context

HOW A 4-MARK ANSWER IS MARKED

A rectangle has length (x + 5) cm and breadth x cm. Its area is 84 cm². Find x.

Strategy $x(x + 5) = 84$, so $x^2 + 5x - 84 = 0$ (1)

Process $(x + 12)(x - 7) = 0$ (1)

Process $x = -12$ or $x = 7$ (1)

Communication x is a length so x > 0: **x = 7** (1)

> **EXAM TIP**
> If you make a mistake early on, keep going: markers follow your working through, so later marks can still be awarded. Never score out working unless you have replaced it.

Working without a calculator (Paper 1)

- Learn your times tables, squares up to 15^2 and cubes up to 5^3. The exact values for 30°, 45° and 60° are useful to know too.
- Multiply decimals by ignoring the point, then put it back: 0.3 × 0.4 = 0.12.
- Divide fractions by multiplying by the reciprocal: $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$.
- Percentages: find 10% and 1%, then build up. 17.5% of 80 = 8 + 4 + 2 = 14.
- Leave answers in surd form or as fractions unless told otherwise, e.g. $3\sqrt{2}$ or $\frac{7}{4}$.

Rounding

Instruction	Example
to 1 decimal place	37.268 → 37.3
to 3 significant figures	0.004 728 → 0.004 73 and 52 670 → 52 700
to the nearest pound	£148.53 → £149
no instruction	give a sensible answer for the context, e.g. 3 significant figures for lengths, 2 decimal places for money

1.1 Surds

Rationalising the denominator

A fraction is in its simplest form when there is no surd on the bottom. To remove √a from the denominator, multiply the top and bottom by √a. This is the same as multiplying by 1, so the value does not change.

WORKED EXAMPLE

Express $\frac{6}{\sqrt{3}}$ with a rational denominator.

Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$

Top and bottom $\frac{6\sqrt{3}}{3}$

Simplify $2\sqrt{3}$

WORKED EXAMPLE

Express $\frac{5}{2\sqrt{10}}$ with a rational denominator.

Multiply by $\frac{\sqrt{10}}{\sqrt{10}}$

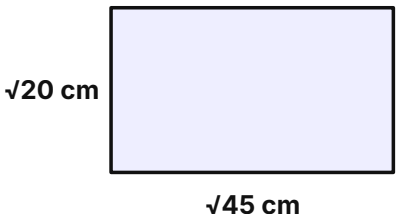
Top and bottom $\frac{5\sqrt{10}}{2 \times 10} = \frac{5\sqrt{10}}{20}$

Simplify $\frac{\sqrt{10}}{4}$

EXAM TIP

Only multiply by the surd part. In $\frac{5}{2\sqrt{10}}$ multiply by $\frac{\sqrt{10}}{\sqrt{10}}$, not by $\frac{2\sqrt{10}}{2\sqrt{10}}$. Then cancel any common factor at the end.

Surds in context



WORKED EXAMPLE

A rectangle is √45 cm by √20 cm. Find its area.

Area $\sqrt{45} \times \sqrt{20} = \sqrt{900}$

Answer **30 cm²**

Quick check

1 Simplify √50.

2 Simplify √48.

3 Simplify √12 + √27.

4 Simplify √5 × √10.

5 Evaluate $(\sqrt{3})^2 + (\sqrt{5})^2$.

6 Express $\frac{4}{\sqrt{2}}$ with a rational denominator.

7 Express $\frac{10}{\sqrt{5}}$ with a rational denominator.

8 Expand and simplify $\sqrt{3}(\sqrt{12} - 1)$.

1.1 Surds

Exam-style questions (15 marks)

1 Simplify $\sqrt{98} + 3\sqrt{2}$. **(non-calculator)**

(2)

2 Express $\frac{12}{\sqrt{6}}$ with a rational denominator. Give your answer in its simplest form. **(non-calculator)**

(2)

3 Expand and simplify $\sqrt{2}(\sqrt{18} + \sqrt{2})$. **(non-calculator)**

(2)

4 A rectangular garden bed at Glen Arrin School measures $\sqrt{12}$ m by $\sqrt{27}$ m. **(non-calculator)**

(a) Calculate the area of the bed. **(2)**

(b) Find the perimeter of the bed. Give your answer as a surd in its simplest form. **(3)**

5 Kai says that $\sqrt{9} + \sqrt{16} = \sqrt{25}$. Is Kai correct? Explain your answer.

(2)

6 Express $\frac{15}{2\sqrt{5}}$ with a rational denominator. Give your answer in its simplest form. **(non-calculator)**

(2)

> **EXAM TIP**
> In surd questions the marks are for the process: show the square factor, the multiplication and the simplified answer. A calculator decimal such as 8.49 gets no marks when the question asks for a surd.

1.2 Indices and scientific notation

In a^n, a is the **base** and n is the **index** (or power). The laws of indices let you simplify expressions without writing out every factor.

Law	Example
$a^m \times a^n = a^{m+n}$	$x^4 \times x^3 = x^7$
$a^m \div a^n = a^{m-n}$	$y^9 \div y^2 = y^7$
$(a^m)^n = a^{mn}$	$(p^3)^4 = p^{12}$
$(ab)^m = a^m b^m$	$(2x)^3 = 8x^3$
$a^0 = 1$	$5^0 = 1$
$a^{-n} = \frac{1}{a^n}$	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
$a^{m/n} = (\sqrt[n]{a})^m$	$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$

KEY POINTS

- Multiply the numbers and add the powers: $3x^4 \times 5x^2 = 15x^6$.
- A negative power means a reciprocal, **not** a negative number.
- For a fractional power, take the root (the bottom number) first, then the power (the top number): $27^{2/3} = 3^2 = 9$.
- 'Positive powers' in the answer means rewrite any negative powers as fractions.

WORKED EXAMPLE

Simplify $6x^5 \times 2x^{-2}$.

Numbers $6 \times 2 = 12$

Powers $5 + (-2) = 3$

Answer $\mathbf{12x^3}$

WORKED EXAMPLE

Evaluate $16^{3/4}$.

Root $\sqrt[4]{16} = 2$

Power $2^3 = 8$

Answer **8**

WORKED EXAMPLE

Expand and simplify $x^{1/2}(x^{3/2} + x^{-1/2})$.

Multiply out $x^{1/2 + 3/2} + x^{1/2 - 1/2}$

Answer $\mathbf{x^2 + 1}$

1.2 Indices and scientific notation

Scientific notation

Very large and very small numbers are written as **$a \times 10^n$** where $1 \le a < 10$ and n is a whole number.

Number	Scientific notation
4 500 000	4.5×10^{6}
0.000 72	7.2×10^{-4}
384 000 000	3.84×10^{8}

Number	To 2 s.f.	To 3 s.f.
34 560	35 000	34 600
0.004 567	0.0046	0.004 57
7.096	7.1	7.10

WORKED EXAMPLE

Calculate $(3.2 \times 10^5) \times (4 \times 10^{-2})$. **(non-calculator)**

Numbers $3.2 \times 4 = 12.8$

Powers $10^{5 + (-2)} = 10^3$

Adjust $12.8 \times 10^3 = \mathbf{1.28 \times 10^4}$

WORKED EXAMPLE

A cell is 2.4×10^{-5} m wide. How many fit across 6×10^{-2} m?

Divide $(6 \times 10^{-2}) \div (2.4 \times 10^{-5})$

Calculator $= 2500$

Answer **2.5×10^3 cells**

Significant figures

Count from the first non-zero digit (see the rounding table above). Zeros at the front of a decimal are not significant; keep place-holding zeros in whole numbers, so 34 560 to 2 s.f. is 35 000, not 35.

> **EXAM TIP**
> On a calculator use the $\times 10^x$ (or EXP) key, not × 10 then ^. Typing 2.4 × 10 ^ −5 without brackets in a division gives the wrong answer.

Quick check

1 Simplify $a^4 \times a^7$.

2 Simplify $(y^3)^4$.

3 Simplify $x^5 \div x^{-2}$.

4 Evaluate $16^{1/2}$.

5 Evaluate $8^{2/3}$.

6 Write 5^{-2} as a fraction.

7 Write 0.000 72 in scientific notation.

8 Round 0.030 49 to 2 significant figures.

9 Calculate $(2 \times 10^3) \times (4 \times 10^5)$.

FREE SAMPLE

National 5 Mathematics Revision Guide

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